2021
(JUNE)

MATHEMATICS
HONOURS

MAT-308

(Real Analysis)

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

1. Answer (a) or (b) **10x1=10**

a) i) State and prove Archimedean propertry. 1=3=4

ii) State and prove Heine-Borel theorem. 1+5=6

b) i) State and prove Bolzano-Weierstrass theorem(for set). 1+5=6

ii) Prove that the union of an arbitrary family of open sets is open. Is arbitrary intersection of open sets necessarily open? Give an example in support of your answer. 4

2. Answer (a) or (b). **10x1=10**

a) i) Prove that the sequence $\{u_n\}$ converges to its supremum if it is bounded and $u_{n+1} \geq u_n \forall n$. Hence or otherwise, show that the sequence $\{u_n\}$ where $u_n = \frac{3}{n+n} + \frac{3}{n+(n-1)} + \cdots + \frac{3}{n+2} + \frac{3}{n+1}$ is convergent. 5

ii) State and prove nested interval theorem. 5

b) i) If f is continuous function in $[a,b]$and $f(a)f(b) < 0$, then there exists a point $c \in\,]a,b[$ such that $f(c) = 0$. Prove the same. 5

ii) Show that if a function f is continuous in $[a,b]$, then it is uniformly continuous in $[a,b]$. 5

3. Answer (a) or (b). **10x1=10**

a) Let $|f(x)| \leq K$ for all x in $[a,b]$ and P be a partition of $[a,b]$ with norm$\leq \delta$. If P^* is a refinement of P containing at most q more points than P, prove that
$\cup(P^*,f) \leq \cup(P,f) \leq \cup(P^*,f) + 2kq\delta$.
Hence prove Darbour's theorem on upper R-integral. 10

b) i) Prove that the oscillation of a bounded function f on an interval $[a,b]$ is the supremum of the set $\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a,b]\}$. 5

ii) Prove that a bounded function f which has only a finite number of points of discontinuity in a closed interval $[a,b]$ is intergrable in $[a,b]$. 5

4. Answer (a) or (b). **10x1=10**

a) i) Test the convergence of $\int_0^1 \log x^7 dx$. 3

ii) Examine the convergence of the improper integral $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ and hence determine the value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$. 7

b) State and prove Frullani's improper integral theorem. By using the same, show that $\int_0^\infty \frac{tan^{-1}ax - tan^{-1}bx}{x}\, dx = \frac{\pi}{2}\log\frac{b}{a}$ 1+6+3=10

5. Answer (a) or (b) **10x1=10**

a) i) Show that the function f, where

$$f(x,y) = \begin{matrix} \frac{xy}{\sqrt{x^2+y^2}}, & \text{if } x^2+y^2 \neq 0 \\ 0 & \text{if } x=y=0 \end{matrix}$$

possesses partial derivative but not differentiable at the origin. 4

ii) State and prove Young's Theorem on the reversal of the order of the partial derivation. 1+5=6

b) i) Show that the function f, where

$$f(x,y) = \begin{cases} \frac{x^3y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

possesses partial derivative but is not differentiable at the origin. 4

ii) State and prove Schwarz's theorem on the reversal of the order of partial derivatives. 1+5=6

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