2021
(JUNE)

MATHEMATICS HONOURS

MAT-309

(Metric Space, Calculus of Variation & Rigid Dynamics)

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

SECTION-A

Answers any five questions: **5 X 5 = 25**

1. Define an open set and prove that a subset G of a metric space X is open if and only if it is a union of open spheres.
2. Let X be a metric space. Then prove that
   a) any union of open sets in X is open
   b) any finite intersection of open sets in X is open.
3. Show that the trivial metric on a set X is a metric i.e. the distinct function $d: X * X \rightarrow R$ defined by
   $$d(a,b) = \begin{cases} 1 & if\ a \neq b \\ 0 & if\ a = b \end{cases}$$
   is a metric.
4. Prove that a subset F of a metric space X is closed if and only its compliment F' is open.
5. State and prove Holder's Inequality.
6. Define a complete metric space. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.
7. State and prove Cantor's Intersection Theorem.
8. Prove that a mapping f from a metric space X into a metric space Y is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.
9. Define the following:
   i) Homeomorphism
   ii) Nowhere dense
   iii) Boundary Point
   iv) Dense Subset
   v) Neighbourhood of a point.
10. Show that a Cauchy sequence is convergent if it has a convergent subsequence.

SECTION-B

Answer any five questions: **5 X 5 = 25**

11. State and prove the Branchistochrone problem in Calculus of Variation.

12. Find the extremals of the functional

$$\int_0^{\pi/2}\left\{2xy+\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right\}dt$$

with conditions $x(0)=0,\qquad x(\pi/2)=-1$

$y(0)=0,\qquad y(\pi/2)=1$

13. State and prove the theorem of parallel axes on a rigid body.

14. State D' Alembert's Principle and discuss the general equation of motion of a rigid body using this principle.

15. If a rigid body swings under gravity from a fixed horizontal axis, show that the time of complete oscillation is

$$2\pi\sqrt{\frac{R^2}{h_g}},$$ where R is its radius

of gyration about the fixed axis and h is the distance between the fixed axis and the centre of inertia of the body.

16. A solid homogeneous cone of height h and vertical angle $2\propto$ oscilates about a horizontal axis through its vertex. Show that the length of simple pendulum is $\frac{1}{5}h(4+tan^2\propto)$.

17. A rough uniform board of mass m and length $2a$ rest on a smooth horizontal plane and a man of mass M walks on it from one end to other. Find the distance through which the board moves in this time.

18. A uniform sphere rolls down an inclined plane rough enough to prevent any sliding, discuss the motion.

19. Show that the momental ellipsoid at the centre of an ellipsoid is $(b^2+c^2)x^2+(a^2+c^2)y^2+(a^2+b^2)z^2=constant.$

20. Find the equation of motion in two dimension when the forces acting on the body are finite.

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