2021
(JUNE)
MATHEMATICS
HONOURS
MAT-310
Special Theory of Relativity & Tensor

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

SECTION-A

Question Nos. 1 & 2 are compulsory and choose any 3 (three) questions from the remaining of Section-A

1. a) The basic theory of field is governed by: 1x5=5
   - i) Laplace Transformation
   - ii) Lorentz Transformation
   - iii) Legendre's Transformation
   - iv) Lagrange's Transformation

   b) Lorentz Transformation reduces to Galilean one if:
   - i) $v = c$
   - ii) $v \ll c$
   - iii) $v \gg c$
   - iv) None of the above.

   c) Aberration of light from stars is caused due to:
   - i) the travelling of light in the atmosphere
   - ii) the scattering of light by the air particles
   - iii) elliptical orbit of the earth around the sun
   - iv) None of the above.

   d) The relation between momentum and energy is:
   - i) $E^2 = p^2 c^2 - m_0^2 c^4$
   - ii) $E^2 = p^2 c^2 - m_0^2 c^2$
   - iii) $E^2 = p^2 c^2 + m_0^2 c^4$
   - iv) $E^2 = p^2 c^2 + m_0^2 c^2$

   e) The resultant of two velocities of light each of which is less than c is also:
   - i) greater than c
   - ii) equal to c
   - iii) $v^2 > c^2$
   - iv) less than c.

2. a) What do you mean by binding energy?   2

   b) Calculate the binding energy of one helium nucleus, if the mass of a hydrogen atom is 1.00814 amu and that of a helium atom is 4.00388 amu.   4

3. a) Prove that $m = \dfrac{m_0}{\left(1 - \dfrac{u^2}{v^2}\right)^{1/2}}$, where the symbols have their usual notations.   5

b) The length of a rocket ship is 100 metres on the ground. When it is in flight its length observed on the ground is 99 metres. Find the speed of the rocket.   3

4. a) Show that the apparent length is contracted by the factor $\sqrt{\left(1 - \dfrac{u^2}{v^2}\right)}$ in the direction of relative

motion $(l' < l)$ 5

b) Show that is $x^2 + y^2 + z^2 - c^2 t^2$ Lorentz invariant.   3

5. a) Prove that $\left(p^2 - \dfrac{E^2}{c^2} \right)$ is Lorentz invariant.   5

b) Suppose the half-life of a certain particle is 10^{-7} second, when it is at rest. What will be its half-life when it is travelling with a speed of $0.99c$?   3

6. a) Deduce the transformation equations for momentum?   5

b) The rest mass of an electron is 9×10^{-7} gm. What will be mass if it were moving with velocity $\dfrac{4}{5}$ times the speed of the light?   3

SECTION-B

Questions No. 7 is compulsory and any 2 (two) questions from the remaining Section-B

7. a) What is the Contravariant tensor of rank one or first order?

b) Define the Kronecker delta.

c) Write the term of the following indicated sums $\dfrac{\delta}{\delta x^k}\left(\sqrt{(g)}A^k\right), N = 3$.

d) Write the law of transformation for the tensor B^k_{jk}.

e) What is symmetric tensor?   1x5=5

8. The outer product of two tensors in a tensor whose rank (or order) is the sum of the ranks of the two tensors.   5

9. A symmetry tensor of rank two has at most $\dfrac{1}{2} N(N+1)$ different components in V_N.   5

10. a) If A_{ij} is a skew-symmetric tensor. Prove that $\left(\delta^i_j \delta^k_l + \delta^i_l \delta^k_j\right)A_{ik} = 0$

b) Suppose g_{ij} and g^{ij} are reciprocal symmetric tensors of the second order.

Then prove that $g^{ij} \dfrac{\delta g_{ij}}{\delta x^k} + g_{ij} \dfrac{\delta g^{ij}}{\delta x^k} = 0$.   3+2=5

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