**2021**
**(JUNE)**

**MATHEMATICS**
**HONOURS**

**MAT-311**

**Mathematical Statistics (Optional)**

**Theory**

**Full Marks: 50**
***The figures in the margin indicates full marks for the questions***
***Answer all the questions.***

1. Obtain a formula for median of Exponential distribution with parameter λ>0. Show that moment generating function for a exponent variate with parameter λ>0 is
$$1+\frac{t}{\lambda}+\left(\frac{t}{\lambda}\right)^2+\cdots$$ 10

**Or**

On x-axis (n+1) points are taken independently between the origin and x=1, all positions being equally likely. Show that probability that the $(k+1)^{th}$ of these points, counted from the origin, lies in the interval $x-\frac{1}{2}dx$ to $x+\frac{1}{2}dx$ is $\binom{n}{k}(n+1)x^k(1-x)^{n-k}dx$. 10

2. If $E(X)=\sum xP_x$ exists prove that $E(X^2)=P''(1)+P'(1)=2Q'(1)+Q(1)$ and $V(X)=2Q'(1)+Q(1)-\{Q(1)\}^2=P''(1)+P'(1)-\{P'(1)\}^2$. 10

**Or**

"A random variable X may have no moments although its moment generating function exists". Justify the above statement with an example. 10

3. Define convergence of random variable in probability. Let $\{X_n\}$ be a sequence of random variable such that $X_n\to X$ and $X_n\to X'$. Prove that $X=X'$. 10

**Or**

Let $\{X_n\}$ be a sequence of random variable such that $X_n \xrightarrow{r} X$ Prove that $E|X_n| \xrightarrow{r} E|X|$. 10

4. Prove that mean=mode= median for normal distribution. 10

**Or**

Draw the normal density curve and give six geometrical significances of this curve. 10

5. State and prove Chebyshev's inequality. 10

**Or**

State and prove De-Moivre-Laplace central limit theorem. 10

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